

# The CKM matrix from anti-SU(7) unification of GUT families

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We estimate the CKM matrix elements in the recently proposed minimal model, anti-SU(7) GUT for the family unification,  $[3] + 2[2] + 8[\bar{1}] + (\text{singlets})$ . It is shown that the real angles of the right-handed unitary matrix diagonalizing the mass matrix can be determined to fit the Particle Data Group data. However, the phase in the right-handed unitary matrix is not constrained very much. We also includes an argument about allocating the Jarlskog phase in the CKM matrix. Phenomenologically, there are three classes of possible parametrizations,  $\delta_{\text{CKM}} = \alpha, \beta$ , or  $\gamma$  of the unitarity triangle. For the choice of  $\delta_{\text{CKM}} = \alpha$ , the phase is close to a maximal one.

PACS numbers: 12.10.Dm, 11.25.Wx, 11.15.Ex

Keywords: Family unification, Anti-SU(7), CKM matrix, Jarlskog phase, GUTs

## I. INTRODUCTION

At present, the unitarity triangle is determined with a very high precision such that any flavor unification models can be tested against it. Therefore, we attempt to see whether the recently proposed unification of grand unification families (UGUTF) based on anti-SU(7) [1] is ruled out or not, from the determination [2] of the Cabibbo-Kobayashi-Maskawa(CKM) matrix elements [3–6]. A simple CKM analysis can be performed in the Kim-Seo(KS) parametrization [7] where only complex phase gives the invariant Jarlskog phase itself [8]. This phase is called the CKM phase  $\delta_{\text{CKM}}$ .

Most family unification models assume a factor group  $G_f$  in addition to the standard model (SM) or grand unification (GUT), where for  $G_f$  continuous symmetries such as SU(2) [9], SU(3) [10], or U(1)'s [11], and discrete symmetries such as  $S_3$  [12],  $A_4$  [13],  $\Delta_{96}$  [14],  $\mathbf{Z}_{12}$  [15] have been considered. However, a true unification of families in the sense that the couplings of the family symmetry are unified with the three gauge couplings of SM has started with the seminal paper by Georgi [16], starting from an SU( $N$ ) GUT [17, 18]. Along this line, a UGUTF based on SU(7)  $\times$  U(1) <sup>$n$</sup>  was suggested [1]. It is derived from string compactification, and contains anti-SU(5) subgroup representations of sixteen chiral fields for one family. These are  $\mathbf{10}_{+1/5} (d^c, u, d, N^0)$ ,  $\bar{\mathbf{5}}_{-3/5} (d^c, \nu_e, e)$ , and  $\mathbf{1}_{+5/5} (e^+)$  [19, 20]. It is comforting that a plethora of anti-SU(5) or flipped-SU(5) GUTs can be derived in string compactifications [21, 22].

The anti-SU(7) solution of the family problem is to put all fermion representations in

$$\Psi^{[ABC]} + 2\Psi^{[AB]} + 8\Psi_{[A]} + \text{singlets} \equiv \mathbf{35} \oplus 2 \times \mathbf{21} \oplus 8 \times \bar{\mathbf{7}} + \mathbf{1}'\text{'s}, \quad (1)$$

where the indices inside square brackets imply anti-symmetric combinations, and the bold-faced numbers are the dimensions of representations. The color indices are 1, 2, 3 and weak indices are 4, 5. With U(1)'s, it is possible to assign the electromagnetic charge  $Q_{\text{em}} = 0$  for separating the color and weak charges at the location [45], which is the key point for realizing the doublet-triplet splitting in the GUT BEH multiplets [1]. The merits of the UGUTF of Ref. [1] are, (i) it allows the missing partner mechanism naturally based on a suitable  $\mu$  parameter [23], (ii) it is obtained from string compactification, and (iii) it leads to plausible Yukawa couplings. The first merit has been already discussed in Ref. [1]. The second merit is the following. The R-parity in SUSY and the Peccei-Quinn symmetry are greatly used for proton longevity and toward a solution of the strong CP problem and cold dark matter [24]. Because of the gravity spoil of such symmetries in general [25, 26], discrete gauge symmetries were considered in the bottom approach [27, 28]. It can be a discrete subgroup of some gauge group. In the top-down approach, such as in models from string compactification, the resulting approximate discrete and global symmetries are automatically allowed since string theory describes gravity without such problems [29, 30].

In this paper, we focus on the third merit by adopting the spectra obtained in Ref. [1], and explicitly calculate the CKM matrix. Here, we do not use the full description of Yukawa couplings dictated from string theory [31], but use

just the supergravity couplings including non-renormalizable terms<sup>1</sup> suppressed by the string scale,  $M_s$ . Thus, every nonrenormalizable term introduces an undetermined coefficient of  $O(1)$ . Here, we reduce the number of couplings, using the  $\mathbf{Z}_{12-I}$  discrete symmetry implied from its origin of  $\mathbf{Z}_{12-I}$  orbifold compactification [1].

## II. SOME COMMENTS RELATED TO THE JARLSKOG DETERMINANT

It is known that  $\delta_{\text{CKM}} \approx 90^\circ$  in the KS parametrization [33]. The Particle Data Group(PDG) compilation gives  $\delta_{\text{CKM}} = (85.4^{+3.9}_{-3.8})^\circ$ , *i.e.* our  $\delta_{\text{CKM}}$  is their  $\alpha_{\text{PDG}}$  [2]. We consider this as a maximal phase with the prescribed real angles. The Jarlskog determinant  $J$  is the area of the Jarlskog parallelogram which has two angles whose sum is  $\pi$ . The area of the parallelogram has the form: (*combination of real angles*)  $\cdot \sin \delta_{\text{CKM}}$ . So, the Jarlskog phase can be taken as  $\delta_{\text{CKM}}$  or  $\pi - \delta_{\text{CKM}}$ . Let us define ‘Jarlskog triangle’ by cutting the parallelogram along a diagonal line, and the Jarlskog invariant phase is the angle opposite to the cutted line. One crucial question is whether the Jarlskog phase is parametrization-independent or not. This is because the Jarlskog determinant, *i.e.* the area, can be made the same in different parametrization schemes by appropriately changing (*combination of real angles*) and  $\sin \delta_{\text{CKM}}$ . We argue that there are only three classes of the CKM parametrizations from length sides of  $O(\lambda^3)$  unitarity triangle. From the unitarity triangle of  $B_s$  meson decay with  $O(\lambda^3)$  lengths, there are three angles  $\alpha, \beta$  and  $\gamma$ , and we can define three classes of parallelograms with the same area. Since there are six different unitarity triangles, three *vertical cases* in choosing two columns and three *horizontal cases* in choosing two rows, the total number of possibilities is 18. Out of these 18 angles, 4 angles (having  $\delta_{\text{CKM}} \simeq 0$ ) with side lengths of  $O(\lambda)$  and  $O(\lambda^2)$  from horizontal and vertical cases are phenomenologically excluded. Furthermore, the invariant phase appears in all six triangles. Since the unitarity triangle of  $B_s$  meson decay is known rather accurately, only three angles are suitable for  $\delta_{\text{CKM}}$ . The KS parametrization uses  $\alpha$  of the unitarity triangle of  $B_s$  meson decay as  $\delta_{\text{CKM}}$  while the Chau-Keung-Maiani (CK) parametrization [6] uses  $\gamma$  as  $\delta_{\text{CKM}}$ . Since  $\alpha \simeq 90^\circ$ , it is minimal (also, see below) to adopt the KS parametrization since there can be one Jarlskog phase  $\frac{\pi}{2}$ . In the other classes, there are two Jarlskog phases,  $\gamma$  and  $\pi - \gamma$ , or  $\beta$  and  $\pi - \beta$ .

If the phase is parametrization-dependent, it is not so important to try to determine very accurately  $\alpha, \beta, \gamma$  in the unitarity triangle of  $B_s$  meson decay in Particle Data Book [2]. Here, we argue that the CKM phase  $\delta_{\text{CKM}}$  is scheme independent up to three classes. Assume that the weak CP violation is introduced spontaneously [34] by a complex vacuum expectation value of the standard model (SM) singlet  $X$  [35]. Suppose the phase of  $\langle X \rangle$  is  $2\pi n/N_{\text{DW}}$  where  $n$  and  $N_{\text{DW}}$  do not have a common divisor. Thus, the vacuum has  $N_{\text{DW}}$  different domains which are separated by domain walls [36]. Depending on the value of  $\delta_{\text{CKM}}$ , we can similarly define the domain wall number of the CKM matrix,  $N_{\text{CKM}}$ . Let the phase  $\delta$  of the SM singlet  $X$  vary continuously from 0 to  $2\pi$ . Along this variation, one passes through the different domains of number  $N_{\text{DW}}$ . Now, suppose we perform weak CP violation experiments looking at the Jarlskog phase. Observe that  $\delta_{\text{CKM}}$  must be proportional to the phase of  $\langle X \rangle$  since there will be no CP violation if  $\langle X \rangle$  is real. In the same domain, measurements on the weak CP phase must be identical. So, we obtain  $N_{\text{CKM}} \leq N_{\text{DW}}$ . In addition, in the two adjacent domains, measurements on weak CP phase must be different, leading to  $N_{\text{CKM}} \geq N_{\text{DW}}$ . Thus, we obtain  $N_{\text{CKM}} = N_{\text{DW}}$ . In this Gedanken experiment with spontaneous CP violation,  $|\delta_{\text{CKM}}|$  is the magnitude of the phase of the VEV  $\langle X \rangle$ . So, it must be scheme independent, up to three classes, since in any CKM parametrization the VEV of fundamental field  $X$  is not introduced. In this argument, it is better to use the parametrization scheme where the phase of  $\langle X \rangle$  sits at origin, regardless of the product of the combinations of real angles. Namely, the phase  $\delta_{\text{CKM}}$  has an invariant meaning, up to three classes. It is a topological argument and may be applicable for the case of complex Yukawa couplings also because one can mimic the complex Yukawa couplings by VEVs of SM singlets. Thus, the first result from the Jarlskog determinant is that the phase  $\delta_{\text{CKM}}$  has a parametrization-independent meaning, up to three classes. The second result is that the product of the lengths of two sides enclosing the Jarlskog phase  $\delta_{\text{CKM}}$  is invariant.

The third aspect is the following. Firstly, as an illustration, consider the Kobayashi-Maskawa(KM) parametrization [4]. Its determinant is not 1, but  $-e^{i\delta'}$ . A proper redefinition, making the determinant real and rotating all six Jarlskog triangles without changing the shapes, is to multiply  $e^{i(\pi-\delta')/3}$  to every element. It is equivalent to multiplying  $e^{i\delta_0}$  ( $\delta_0 = (\pi - \delta')/3$ ) to  $\bar{u}_L, \bar{c}_L$ , and  $\bar{t}_L$  fields such that the newly defined primed fields are  $\bar{u}'_L = e^{i\delta_0} \bar{u}_L$ ,  $\bar{c}'_L = e^{i\delta_0} \bar{c}_L$ , and  $\bar{t}'_L = e^{i\delta_0} \bar{t}_L$ . Then, obviously the shapes of all six triangles are not changed. But this introduces a factor  $e^{i\pi/3}$  in every elements. To keep the shapes of at least three *vertical* Jarlskog triangles, including the familiar one in the PDG

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<sup>1</sup> String calculation of all non-renormalizable couplings are not available at present. See, for an attempt, Ref. [32].

book, multiply  $\text{diag.}(1, 1, -e^{-i\delta'})$  on the right-hand side of the KM matrix, leading to

$$V'_{\text{KM}} = \begin{pmatrix} c_1, & -s_1 c_3, & e^{-i\delta'} s_1 s_3 \\ c_2 s_1, & -e^{i\delta'} s_2 s_3 + c_1 c_2 c_3, & -s_2 c_3 - e^{-i\delta'} c_1 c_2 s_3 \\ s_1 s_2, & c_2 s_3 e^{i\delta'} + c_1 s_2 c_3, & c_2 c_3 - e^{-i\delta'} c_1 s_2 s_3 \end{pmatrix} \quad (2)$$

from which we obtain

$$\begin{aligned} \alpha &= \text{Arg.} \left( -\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right) = \text{Arg.} \left( \sin \theta_2 (-e^{-i\delta'} \cos \theta_2 \cot \theta_3 \sec \theta_1 + \sin \theta_2) \right), \\ \beta &= \text{Arg.} \left( -\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right) = \text{Arg.} \left( \frac{\cos \theta_2 (\cos \theta_3 + e^{i\delta'} \cos \theta_1 \cot \theta_2 \sin \theta_3)}{\cos \theta_2 \cos \theta_3 - e^{i\delta'} \cos \theta_1 \sin \theta_2 \sin \theta_3} \right), \\ \gamma &= \text{Arg.} \left( -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right) = \text{Arg.} \left( \frac{\cos \theta_1 \sec \theta_2 \sin \theta_3}{e^{-i\delta'} \cos \theta_3 \sin \theta_2 + \cos \theta_1 \cos \theta_2 \sin \theta_3} \right). \end{aligned} \quad (3)$$

Note that  $\alpha \simeq \pi - \delta'$ , *i.e.* the KM parametrization uses  $\alpha$  of PDG as  $\delta_{\text{CKM}}$ .<sup>2</sup>

It is very useful if the CKM matrix itself contains the invariant phase  $\delta_{\text{CKM}}$  in a visible manner.  $J$  is always arising with  $O(\lambda^3)$  multiplied. Since the (13) and (31) elements of  $V_{\text{CKM}}$  are already  $O(\lambda^3)$  [5], it is convenient for the phase  $e^{i\delta_{\text{CKM}}}$  to appear either in the (31) element or in the (13) element with one row or one column real. Since the (22) element is an almost real  $O(1)$  constant, it will lead to  $J = \text{Im } V_{31}^* V_{22}^* V_{13}^* = O(\lambda^6) \sin \delta_{\text{CKM}}$  [33].<sup>3</sup> It is convenient to make the first row real, *i.e.* the (13) element real. Then, the denominator  $V_{ud} V_{ub}^*$  in Eq. (3) is real and the Jarlskog triangle has one side on  $x$ -axis. The angle at the origin is  $\delta_{\text{CKM}}$ . On the other hand, the CK parametrization [6] has real values for both the first row and first column, and its determinant = 1. Thus,  $J$  must be contributed from the phase in the (22) element. The  $V_{(cd)} \cdot V_{(cb)}^*$  component (for the (22) element, or the  $(cc)$  element in the mass eigenstate bases) appears for  $\beta$  and for  $\gamma$  in Eq. (3). For the one in the numerator, *i.e.* in  $\beta$ ,  $V_{tb}^*$  in the denominator is also complex, and  $V_{(cd)} \cdot V_{(cb)}^*$  alone cannot determine  $\delta_{\text{CKM}}$ . On the other hand, the one in the numerator, *i.e.* in  $\gamma$ , the numerator is real, and  $V_{(cd)} \cdot V_{(cb)}^*$  alone determines  $\delta_{\text{CKM}}$ . Thus,  $\delta_{\text{CKM}}$  is  $\gamma$  in the CK parametrization. We can generalize this statement. Let us use the parametrizations such that the large components of the diagonal elements are real. Then, if the first row or first column is real,  $\delta_{\text{CKM}} = \alpha$ . If both the first row and first column are real, then  $\delta_{\text{CKM}} = \gamma$ . To have  $\beta$  as  $\delta_{\text{CKM}}$ , we need that the (22) element contains a large imaginary part. In this analysis, it was useful to remember the formula of Ref. [33]:  $J = \text{Im } V_{31}^* V_{22}^* V_{13}^*$ .

The invariant Jarlskog phase appears in all Jarlskog triangles, not necessarily at the origin. Let us take, as an illustration purpose,  $\alpha \simeq 90^\circ = \frac{2\pi}{4}$ ,  $\beta \simeq 22.5^\circ = \frac{2\pi}{16}$ , and  $\gamma \simeq 67.5^\circ = \frac{2\pi}{16} \times 3$  which are within the experimental bounds. If these phases appear from some  $\mathbf{Z}_N$  symmetry, we can choose three kinds of  $N$  depending on which angle is used for  $\delta_{\text{CKM}}$ . If  $\alpha, \beta$ , and  $\gamma$  are used for  $\delta_{\text{CKM}}$ ,  $N_{\text{DW}}$  of  $\langle X \rangle$  must be 4, 16, and 16, respectively. In this paper we use the KS parametrization [7, 33], which is a kind of minimal one,

$$V_{\text{KS}} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -c_2 s_1 & e^{-i\delta_{\text{CKM}}} s_2 s_3 + c_1 c_2 c_3 & -e^{-i\delta_{\text{CKM}}} s_2 c_3 + c_1 c_2 s_3 \\ -e^{i\delta_{\text{CKM}}} s_1 s_2 & -c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\text{CKM}}} & c_2 c_3 + c_1 s_2 s_3 e^{i\delta_{\text{CKM}}} \end{pmatrix}. \quad (4)$$

Note that  $J$  is given as,  $J_{\text{KS}} = \text{Im } V_{31}^* V_{22}^* V_{13}^* = c_1 c_2 c_3 s_1^2 s_2 s_3 \sin \alpha = O(\lambda^6 - \lambda^7)$  in the KS parametrization and  $J_{\text{CK}} = \text{Im } V_{31}^* V_{22}^* V_{13}^* = c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23} \sin \gamma = O(\lambda^6 - \lambda^7)$  in the CK parametrization. If the Cabibbo angle  $\theta_C = s_1 c_3 = s_{12} c_{13}$  is fixed,  $J/\sin \theta_C = c_1 c_2 s_1 s_2 s_3 \sin \alpha = c_{12} c_{13} c_{23} s_{13} s_{23} \sin \gamma$ . For a numerical study, we can choose a *vertical* Jarlskog triangle of the first and second columns, where two  $O(\lambda)$  side lengths are  $|c_1 c_3 s_1|, |c_2 s_1 (c_1 c_2 c_3 + s_2 s_3 e^{-i\alpha})|$ , and an  $O(\lambda^5)$  side length is  $e^{i\alpha} s_1 s_2 |(c_1 c_3 s_2 - c_2 s_3 e^{-i\alpha})|$  with the phase explicitly written for the  $O(\lambda^5)$  side to rotate it freely. The corrected area depending on  $\theta_2, \theta_3$  and  $\alpha$  is  $J/c_1 \sin^2 \theta_C = \frac{1}{2} \sin(2\theta_2) \tan(\theta_3) \sin \alpha$ . For given  $\sin 2\theta_2$  and  $\tan \theta_3$ , we can rotate  $\alpha$  to  $90^\circ$  to obtain the largest  $\delta_{\text{CKM}}$  since in our choice of  $\alpha \sim 90^\circ$  is allowed. We cannot give this argument for  $\delta_{\text{CKM}} = \gamma$ , where  $\gamma$  is far from  $90^\circ$ .

It is pointed out that if  $\delta_{\text{CKM}} = \pm \delta_{\text{PMNS}}$  is empirically proved then the idea of spontaneous CP violation à la Froggatt and Nielsen with a UGUTF makes sense [38]. In this case, the value  $\delta_{\text{PMNS}}$  will choose one class of the CKM parametrizations we discussed here.

<sup>2</sup> Note that  $\delta_{\text{CKM}}$  is defined  $\alpha$  or  $\pi - \alpha$ , depending on the cutted diagonal line.

<sup>3</sup> This form is true in any parametrization with  $\text{Det. } V_{\text{CKM}} = 1$ .

### III. YUKAWA COUPLINGS AND MASSES

#### A. U(1) charges in anti-SU(7)

To check the Yukawa couplings, it is useful to have U(1) charges in the anti-SU(7) model. For completeness, therefore, we list them. For the fundamental representation **7**, the U(1) charges belonging to SU(5) and SU(7) are defined as

$$\begin{aligned} X_5 &= \left( \frac{2}{30}, \frac{2}{30}, \frac{2}{30}, \frac{-3}{30}, \frac{-3}{30}, 0, 0 \right) \\ Z_7 &= \left( \frac{-2}{7}, \frac{-2}{7}, \frac{-2}{7}, \frac{-2}{7}, \frac{-2}{7}, \frac{5}{7}, \frac{5}{7} \right) \end{aligned} \quad (5)$$

The extra U(1) charge beyond SU(7) is

$$Z = \left( \frac{-5}{7}, \frac{-5}{7}, \frac{-5}{7}, \frac{-5}{7}, \frac{-5}{7}, \frac{-5}{7}, \frac{-5}{7} \right). \quad (6)$$

For the matter **7**, therefore, we represent it as  $\mathbf{7}_{-5/7}$ . The electroweak hypercharge  $Y$  of the SM and the U(1) charge  $X$  of the flipped-SU(5) are defined as

$$\begin{aligned} Y &= \left( \frac{-1}{3}, \frac{-1}{3}, \frac{-1}{3}, \frac{1}{2}, \frac{1}{2}, 0, 0 \right) = X_5 + X, \\ X &= \left( \frac{18}{30}, \frac{18}{30}, \frac{18}{30}, \frac{18}{30}, \frac{18}{30}, 0, 0 \right) = -\frac{3}{5}(Z_7 + Z), \end{aligned} \quad (7)$$

When **21** branches to SU(5) representations **10**,  $2 \cdot \mathbf{5}$ , and **1**, the SM U(1) charges are required to be the familiar ones, determining subscripts  $a, b, c$  in the following ,

$$(\mathbf{10}_a; \mathbf{5}_b, \mathbf{5}_b; \mathbf{1}_c) = \left( \frac{1}{3}(d^c), \frac{1}{6}(q), 0(N); \mathbf{5}_a, \mathbf{5}_a, \mathbf{1}_b \right) \rightarrow a = \frac{1}{5} (\sim \frac{6}{5}), b = \frac{3}{5}, c = 0, \quad (8)$$

where we used Eqs. (5) and (7) and used quantum numbers of  $\mathbf{21} = \Psi^{[AB]}$ . When **35** branches to SU(5) representations as  $\overline{\mathbf{10}}$ ,  $2 \cdot \mathbf{10}$ , and **5**, similarly subscripts  $d, e, f$  in the following are determined as

$$\begin{aligned} (\overline{\mathbf{10}}_d; \mathbf{10}_e, \mathbf{10}_e; \mathbf{5}_f) &= \left( \frac{-1}{3}, \frac{-1}{6}, 0(\overline{N}); 2 \left[ \frac{1}{3}(d^c), \frac{1}{6}(q), 0(N) \right]; \mathbf{5}_g \right) \\ &\rightarrow d = \frac{-1}{5} (\sim \frac{9}{5}), e = \frac{1}{5} (\sim \frac{6}{5}), f = \frac{3}{5}, \end{aligned} \quad (9)$$

where we used Eqs. (5) and (7) and used quantum numbers of  $\mathbf{35} = \Psi^{[ABC]}$ . Because of the compact group nature, the naive U(1) charge calculation given in the bracket just by the tensor representation components is not exact. We use the  $|X| \leq 1$  for the fundamental representation Eq. (7). With two SU(5) indices, the  $|X|$  charge are redundantly added, and we subtract  $\pm 1$ . With one more indices in addition to the two indices, again we subtract  $\pm 1$  once more. The rule to use in Eqs. (8) and (9) is to subtract  $(N-1)$  from  $X$  for  $N$  SU(5) indices. Because  $d = -\frac{1}{5}$  and  $e = \frac{1}{5}$ , one vectorlike pair of **10** and  $\overline{\mathbf{10}}$  are removed at the GUT scale and we obtain two  $\mathbf{10}_{1/5}$ 's from two **21** of Eq. (8) and one  $\mathbf{10}_{1/5}$  from **35** of Eq. (9). In particular, note that  $\overline{\mathbf{10}}_{-1/5}$  of Eq. (9) contains  $\overline{N}$  which can develop a VEV. Thus, there result three SM families. Therefore, for the chiral representations we treat the anti-SU(5) representations as usual. For the BEH scalars, we need U(1) charges of the anti-SU(5) as  $\mathbf{5}_{-2/5}$  which houses  $H_d$  and  $\mathbf{5}_{+2/5}$  which houses  $H_u$ .

Now we can calculate the Yukawa coupling matrices for the quark sector. Here, we attempt to calculate  $V_{\text{CKM}}$ , and comment on  $U_{\text{PMNS}}$  in the end. For charged leptons including  $e^+, \mu^+, \tau^+$ , which appear as SU(7) singlets, we must obtain all SU(7) singlet spectra. These singlets are not available at present. Thus, we try to calculate  $V_{\text{CKM}}$  and  $U_{\text{PMNS}}$  without the knowledge on the singlets. The CKM matrix is obtained if we know the  $Q_{\text{em}} = \frac{2}{3}$  and  $-\frac{1}{3}$  quark mass matrices,

$$\begin{aligned} \overline{u}_L M^{(2/3)} u_R &= \mathbf{10}_{1/5} \overline{\mathbf{5}}_{-3/5} \langle \overline{\Phi}_{\text{BEH}, 2/5} \cdot (\cdots) \rangle \\ \overline{d}_L M^{(-1/3)} d_R &= \mathbf{10}_{1/5} \mathbf{10}_{1/5} \langle \Phi_{\text{BEH}, -2/5} \cdot (\cdots) \rangle \end{aligned} \quad (10)$$

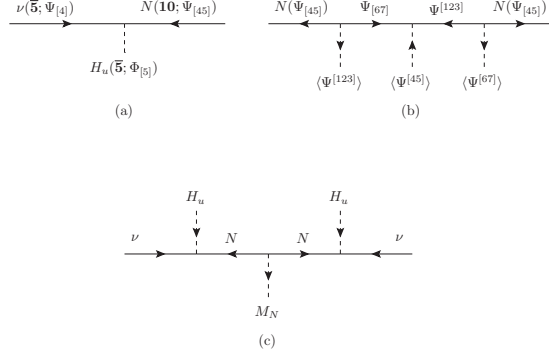


FIG. 1: The double seesaw with (a) The Dirac mass and (b) the Majorana mass of  $N$ , and (c) the seesaw mass of the SM neutrinos.

where we used the anti-SU(5) notation. For the PMNS matrix, we do not need information on the Yukawa couplings of the charged leptons. On the other hand, we need to know

$$\mathbf{10}_{1/5} \bar{\mathbf{5}}_{-3/5} \langle \bar{\Phi}_{\text{BEH},2/5} \cdot (\cdots) \rangle \quad (11)$$

for the Dirac mass of  $N_i$  (the (45) element of  $\mathbf{10}$ ) and  $\nu_j$  and  $N_i - N_k$  masses. The Dirac mass coupling is shown in Fig. 1(a) and the Majorana mass term of  $N$  is shown in Fig. 1(b). The seesaw is a double seesaw as depicted in Fig. 1(c), which is obtained from Fig. 1(a) and Fig. 1(b). But, we do not need the charged lepton mass matrices.

As commented in Ref. [1], the  $b$ -quark mass is expected to be much smaller than the  $t$ -quark mass,  $O(\langle T_{3,\text{BEH}}^{\mathbf{21}} \rangle \langle T_{3,\text{BEH}}^{\mathbf{7}} \rangle / M_s \langle T_{6,\text{BEH}}^{\mathbf{7}} \rangle)$ , where  $\langle T_{3,\text{BEH}}^{\mathbf{21}} \rangle$  is the SU(5) splitting VEV  $\langle \Phi^{[67]} \rangle$ . Thus, we expect  $m_b/m_t \sim \frac{\langle \Phi^{[67]} \rangle}{M_s \tan \beta}$ . Even if  $\tan \beta = O(1)$ , we can fit  $m_b/m_t$  to the observed value by appropriately tuning  $\langle \Phi^{[67]} \rangle$ . A similar suppression occurs for the second family members.

### B. A democratic submatrix of $M_{\text{weak}}$

The multiplicity 2 of the fields from  $T_3$  leads to a democratic form for the submatrix of the mass matrix. Thus, we consider

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (12)$$

which can be diagonalized to give the eigenvalues 0 and 1. The democratic form can be extended to have a permutation symmetric form  $S_2$  which has only singlet representations. Introducing two small numbers  $x$  and  $y$  (for the two independent singlets) for breaking the  $S_2$  symmetry, it can be diagonalized to

$$M = \begin{pmatrix} \frac{1}{2} + \frac{y}{2}, & \frac{1}{2} + \frac{x}{2} \\ \frac{1}{2} + \frac{x}{2}, & \frac{1}{2} + \frac{y}{2} \end{pmatrix} \rightarrow \begin{pmatrix} -\frac{x+y}{2}, & 0 \\ 0, & 1 + \frac{x+y}{2} \end{pmatrix} \rightarrow \begin{pmatrix} -\epsilon + \epsilon', & 0 \\ 0, & 1 + \epsilon \end{pmatrix}, \text{ with } \epsilon = \frac{x+y}{2}, \epsilon' = y. \quad (13)$$

by

$$U_{2 \times 2}^\dagger M U_{2 \times 2}, \text{ with } U_{2 \times 2} = \begin{pmatrix} \frac{1}{\sqrt{2}}, & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}, & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (14)$$

A  $3 \times 3$  mass matrix is changed, using a  $U_{3 \times 3}$  matrix,

$$U_{3 \times 3}^\dagger \begin{pmatrix} u_1, & u_2, & u_2 \\ u_3^*, & \frac{1}{2} + \frac{y}{2}, & \frac{1}{2} + \frac{x}{2} \\ u_3^*, & \frac{1}{2} + \frac{x}{2}, & \frac{1}{2} + \frac{y}{2} \end{pmatrix} U_{3 \times 3} \rightarrow \begin{pmatrix} u_1, & 0, & \epsilon_2 \\ 0, & -\frac{x}{2}, & 0 \\ \epsilon_3^*, & 0, & 1 + \frac{x}{2} \end{pmatrix} \quad (15)$$

where  $U_{3 \times 3}$  contains the  $U_{2 \times 2}$  submatrix. Here,  $u$ 's denote small parameters, breaking  $S_2$  spontaneously by the GUT scale VEVs of some SM singlet fields:  $u = O(\langle \Phi \rangle / M_s)$ . In view of the worry on the gravity spoil of discrete symmetries [26–29], two singlet fields are better to be two components of a doublet representation  $\Phi$  of a hypothetical gauge group  $SU(2)$  in the bottom-up scenario.<sup>4</sup> The VEV  $\langle \Phi \rangle$  breaks the  $S_2$  symmetry spontaneously [29]. Then, the trace of  $\Phi$  quantum number is zero. Thus, trace of Eq. (13) is 1, leading to  $\epsilon' = 0$ . Thus, for the gravity-safe correction, which is our case arising from string compactification, let us diagonalize the democratic form to

$$\begin{pmatrix} -\frac{x}{2}, & 0 \\ 0, & 1 + \frac{x}{2} \end{pmatrix}. \quad (16)$$

Therefore, from the information on the origin of families in the untwisted and twisted sectors ( $U_1, T_3, T_5^+$ ) [1], we can write the up- and down-type mass matrices as

$$\frac{M^{(u)}}{m_t} \approx \left( \begin{array}{c|ccc} & \Psi_{[A]}(T_5^+) & \Psi_{[A]}(T_3) & \Psi_{[A]}(T_3) \\ \hline \Psi^{[ABC]}(U_1) & \epsilon_u & 0 & \epsilon_2 \\ \Psi^{[AB]}(T_3) & 0 & x_c & 0 \\ \Psi^{[AB]}(T_3) & \epsilon_3^* & 0 & 1 \end{array} \right) \quad (17)$$

$$\frac{M^{(d)}}{m_b} \approx \left( \begin{array}{c|ccc} & \Psi^{[ABC]}(U_1) & \Psi^{[AB]}(T_3) & \Psi^{[AB]}(T_3) \\ \hline \Psi^{[ABC]}(U_1) & \epsilon_d & 0 & \epsilon_1 \\ \Psi^{[AB]}(T_3) & 0 & x_s & 0 \\ \Psi^{[AB]}(T_3) & \epsilon_1 & 0 & 1 \end{array} \right) \quad (18)$$

where the parameters in Eqs. (17,18) can be complex in general. Note that  $M^{(u)}$  is not a Hermitian matrix and  $M^{(d)}$  is a symmetric matrix. In the bases where Eqs. (17, 18) are written, we proceed to calculate the CKM and PMNS matrices. Parameter  $\epsilon_1$  is given in the democratic form of the  $2 \times 2$  matrix. But Eq. (18) is written in the bases where the democratic form is broken. Thus, we expect two parameters  $\epsilon_1(1 \pm O(x_s))$ . Since  $x_s$  is small, we neglect this  $S_2$  breaking correction. Similar comments apply to  $\epsilon_2$  and  $\epsilon_3$ .

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<sup>4</sup> In the top-down scenario, there will be no gravity spoil problem, presumably satisfying the above condition automatically.

### C. The CKM matrix

Since  $M^{(d)}$  is symmetric, let us absorb two phases  $\epsilon_1$  and  $\epsilon_d$  in  $\Psi^{[AB]}(T_3)$  and  $\Psi^{[ABC]}(T_3)$ . So, the  $d$ -quark Yukawa couplings can be considered real. And we allow a real VEV for  $H_d^0$ . If it were complex, its phase can be absorbed to right-handed  $d$  quarks. Then the real symmetric matrix  $M^{(d)}$  is diagonalized by an orthogonal matrix  $O = O_L = O_R$ ,

$$M_{\text{weak}}^{(d)} = O \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} O^T \quad (19)$$

where

$$M_{\text{weak}}^{(d)} = \begin{pmatrix} m_d c_1^2 + m_s c_2^2 s_1^2 + m_b s_1^2 s_2^2, & m_d c_1 c_3 s_1 - m_s c_2 s_1 [c_1 c_2 c_3 + s_2 s_3] - m_b s_1 s_2 [-c_2 s_3 + c_1 c_3 s_2], & m_d c_1 s_1 s_3 - m_s c_2 s_1 [c_1 c_2 s_3 - s_2 c_3] - m_b s_1 s_2 [c_2 c_3 + c_1 s_2 s_3] \\ m_d c_1 c_3 s_1 - m_s c_2 s_1 [c_1 c_2 c_3 + s_2 s_3] - m_b s_1 s_2 [-c_2 s_3 + c_1 s_2 c_3], & m_d c_3^2 s_1^2 + m_s [c_1 c_2 c_3 + s_2 s_3]^2 + m_b [-c_2 s_3 + c_1 s_2 c_3]^2, & m_d c_3 s_1^2 s_3 + m_s [c_1 c_2 c_3 + s_2 s_3] \cdot [c_1 c_2 s_3 - s_2 c_3] + m_b [-c_2 s_3 + c_1 s_2 c_3] \cdot [c_2 c_3 + c_1 s_2 s_3] \\ m_d c_1 s_1 s_3 - m_s c_2 s_1 [c_1 c_2 s_3 - s_2 c_3] - m_b s_1 s_2 [c_2 c_3 + c_1 s_2 s_3], & m_d c_3 s_1^2 s_3 + m_s [c_1 c_2 s_3 - s_2 c_3] \cdot [c_1 c_2 c_3 + s_2 s_3] + m_b [c_2 c_3 + c_1 s_2 s_3] \cdot [-c_2 s_3 + c_1 s_2 c_3], & m_d s_1^2 s_3^2 + m_s [c_1 c_2 s_3 - s_2 c_3]^2 + m_b [c_2 c_3 + c_1 s_2 s_3]^2 \end{pmatrix} \quad (20)$$

where  $\theta_i$  represent the orthogonal matrix angles  $\theta_{O,i}$ . Here,  $O$  is taken as a real KS parametrization,

$$V_{\text{real}}^{\text{KS}} = \begin{pmatrix} c_{O,1} & s_{O,1} c_{O,3} & s_{O,1} s_{O,3} \\ -c_{O,2} s_{O,1} & s_{O,2} s_{O,3} + c_{O,1} c_{O,2} c_{O,3} & -s_{O,2} c_{O,3} + c_{O,1} c_{O,2} s_{O,3} \\ -s_{O,1} s_{O,2} & -c_{O,2} s_{O,3} + c_{O,1} s_{O,2} c_{O,3} & c_{O,2} c_{O,3} + c_{O,1} s_{O,2} s_{O,3} \end{pmatrix}. \quad (21)$$

We consider  $m_d = O(\lambda^4) \times m_b$ . In Eq. (20), the (23) and (32) elements are vanishing up to  $O(\lambda^9)$  for

$$s_{O,1} = 0, \quad t_{O,2} = t_{O,3}. \quad (22)$$

where the angles are in the 1st quadrant. Angles given in (22) matches to Eq. (18). Thus, there is one angle parameter in  $V_{\text{real}}^{\text{KS}}$ , which is taken as  $\theta_O = \theta_{O,2} = \theta_{O,3}$ . So, the orthogonal matrix diagonalizing  $M^{(d)}$  is

$$V_L^{(d)} = V_R^{(d)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (23)$$

However, because of the  $S_2$  breaking effect as commented above,  $V_{L,R}^{(d)}$  contains small parameters of  $O(\epsilon_1 x_s)$ . For simplicity, we neglect the  $O(\epsilon_1 x_s)$  correction. In the model of Ref. [1],  $\epsilon_1 = O(V_{\text{GUT}}/M_s)$ . This is because one may consider the following for  $\epsilon_1$

$$\frac{1}{M_s^2} \epsilon_{ABCDEFG} \Psi_{U_1}^{[ABC]} \Psi_{T_3}^{[DE]} \Phi_{T_3, \text{BEH}}^F \langle \Phi_{T_3, \text{BEH}}^G \rangle \langle \mathbf{1}_{T_{11}, \text{BEH}} \rangle,$$

and  $m_b = O(V_{\text{GUT}}/M_s)$ . Thus,  $\epsilon_1 x_s$  is estimated to be  $O(\lambda^4)$ . Then, the determination of the CKM matrix depends approximately on the diagonalization of  $M^{(u)}$ .

### D. The CKM and PMNS matrices from anti-SU(7) UGUTF

Now the CKM matrix is determined from the diagonalization of  $M^{(u)}$  by bi-unitary matrices:  $V_L^{(u)}$  and  $V_R^{(u)}$  with  $V_L^{(u)} \neq V_R^{(u)}$ ,

$$V_{\text{CKM}} = V_L^{(u)} O_L^{(d)T} \simeq V_L^{(u)} \quad (24)$$

which does not depend on  $V_R^{(u)}$ . The matrix elements and  $Q_{\text{em}} = \frac{2}{3}$  quark masses have the following relations

$$\begin{aligned} u_L^{(\text{mass } i)} &= V_L^{ia} u_L^a, \quad u_R^{(\text{mass } i)} = V_R^{ia} u_R^a, \\ \bar{u}_R^b M_{\text{weak},u}^{ba} u_L^a &= \bar{u}_R^{(\text{mass } j)} (V_R)^{jb} M_{\text{weak},u}^{ba} (V_L^\dagger)^{ai} u_L^{(\text{mass } i)}. \end{aligned} \quad (25)$$

Thus, the mass matrix elements in the weak basis are

$$M_{\text{weak},u}^{ba} = (V_R^\dagger)^{bj} M_{\text{diag},u}^{ji} (V_L)^{ia} = m_u (V_R^\dagger)^{b1} (V_L)^{1a} + m_c (V_R^\dagger)^{b2} (V_L)^{2a} + m_t (V_R^\dagger)^{b3} (V_L)^{3a}, \quad (26)$$

or

$$\begin{pmatrix} m_u c'_1 c_1 + m_c c'_2 s'_1 c_2 s_1 + m_t s'_1 s'_2 s_1 s_2 e^{i\delta_{\text{CKM}} - i\delta'_{\text{CKM}}} & m_u c'_1 s_1 c_3 - m_c c'_2 s'_1 [c_1 c_2 c_3 + s_2 s_3 e^{-i\delta_{\text{CKM}}}] - m_t s'_1 s'_2 e^{-i\delta'_{\text{CKM}}} \cdot [-c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\text{CKM}}}] & m_u c'_1 s_1 s_3 - m_c c'_2 s'_1 [c_1 c_2 s_3 - s_2 c_3 e^{-i\delta_{\text{CKM}}}] - m_t s'_1 s'_2 e^{-i\delta'_{\text{CKM}}} \cdot [c_2 c_3 + c_1 s_2 s_3 e^{i\delta_{\text{CKM}}}] \\ m_u c'_3 s'_1 c_1 - m_c [c'_1 c'_2 c'_3 + s'_2 s'_3 e^{i\delta'_{\text{CKM}}}] c_2 s_1 - m_t [-c'_2 s'_3 + c'_1 s'_2 c'_3 e^{-i\delta'_{\text{CKM}}}] \cdot s_1 s_2 e^{i\delta_{\text{CKM}}} & m_u c'_3 s'_1 s_1 c_3 + m_c [c'_1 c'_2 c'_3 + s'_2 s'_3 e^{i\delta'_{\text{CKM}}}] \cdot [c_1 c_2 c_3 + s_2 s_3 e^{-i\delta_{\text{CKM}}}] + m_t [-c'_2 s'_3 + c'_1 s'_2 c'_3 e^{-i\delta'_{\text{CKM}}}] \cdot [-c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\text{CKM}}}] & m_u c'_3 s'_1 s_1 s_3 + m_c [c'_1 c'_2 c'_3 + s'_2 s'_3 e^{i\delta'_{\text{CKM}}}] \cdot [c_1 c_2 s_3 - s_2 c_3 e^{-i\delta_{\text{CKM}}}] + m_t [-c'_2 s'_3 + c'_1 s'_2 c'_3 e^{-i\delta'_{\text{CKM}}}] \cdot [c_2 c_3 + c_1 s_2 s_3 e^{i\delta_{\text{CKM}}}] \\ m_u s'_1 s'_3 c_1 - m_c [c'_1 c'_2 s'_3 - s'_2 c'_3 e^{i\delta'_{\text{CKM}}}] c_2 s_1 - m_t [c'_2 c'_3 + c'_1 s'_2 s'_3 e^{-i\delta'_{\text{CKM}}}] \cdot s_1 s_2 e^{i\delta_{\text{CKM}}} & m_u s'_1 s'_3 s_1 c_3 + m_c [c'_1 c'_2 s'_3 - s'_2 c'_3 e^{i\delta'_{\text{CKM}}}] \cdot [c_1 c_2 c_3 + s_2 s_3 e^{-i\delta_{\text{CKM}}}] + m_t [c'_2 c'_3 + c'_1 s'_2 s'_3 e^{-i\delta'_{\text{CKM}}}] \cdot [-c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\text{CKM}}}] & m_u s'_1 s'_3 s_1 s_3 + m_c [c'_1 c'_2 s'_3 - s'_2 c'_3 e^{i\delta'_{\text{CKM}}}] \cdot [c_1 c_2 s_3 - s_2 c_3 e^{-i\delta_{\text{CKM}}}] + m_t [c'_2 c'_3 + c'_1 s'_2 s'_3 e^{-i\delta'_{\text{CKM}}}] \cdot [c_2 c_3 + c_1 s_2 s_3 e^{i\delta_{\text{CKM}}}] \end{pmatrix} \quad (27)$$

where angles in  $V_L^{(u)}$  are  $\theta_i, \delta$  and angles in  $V_R^{(u)}$  are  $\theta'_i, \delta'$ .

Comparing Eqs. (17) and (27), we have 9 constraints. The order of magnitudes of the elements are such that the determinant of mass matrix is  $O(\lambda^8 m_t^3)$  with  $m_c = O(\lambda^2) m_t$  and  $m_u = O(\lambda^6) m_t$ . Thus, the product of (11), (23), and (32) elements is  $O(\lambda^8 m_t^3)$ , and the product of (11), (22), and (33) elements is also  $O(\lambda^8 m_t^3)$ . So, let the (22) element is  $O(\lambda m_t)$  or  $O(\lambda^2 m_t)$ . For  $M_{(11)}^{(u)} = O(\lambda^5 m_t)$ , we require  $M_{(23)}^{(u)} = O(\lambda^{3/2} m_t)$  and  $M_{(32)}^{(u)} = O(\lambda^{3/2} m_t)$ . For  $M_{(11)}^{(u)} = O(\lambda^4 m_t)$ , we require  $M_{(23)}^{(u)} = O(\lambda^2 m_t)$  and  $M_{(32)}^{(u)} = O(\lambda^2 m_t)$ . So, whether the (22) element is  $O(\lambda m_t)$  or  $O(\lambda^2 m_t)$ , we consider  $M_{(23)}^{(u)} = O(\lambda^2 m_t)$  and  $M_{(32)}^{(u)} = O(\lambda^2 m_t)$ . Because the (33) element is  $O(1)$ , we require both (12) and (21) elements to be  $O(\lambda^4)$ . By the same argument, we require both (13) and (31) elements to be  $O(\lambda^3)$ . Thus, we require

$$(11) \lesssim O(\lambda^4) m_t \quad (28)$$

$$(12) \lesssim O(\lambda^4) m_t \quad (29)$$

$$(13) = O(\lambda^3) m_t, \quad (30)$$

$$(21) \lesssim O(\lambda^4) m_t \quad (31)$$

$$(31) = O(\lambda^3) m_t, \quad (32)$$

$$(22) \lesssim O(\lambda^2 \text{ or } \lambda) m_t \quad (33)$$

$$(23) = O(\lambda^2) m_t \quad (34)$$

$$(32) = O(\lambda^2) m_t \quad (35)$$

$$(33) = O(1) m_t \quad (36)$$

where we used  $m_t = 173.21$ ,  $m_c = 1.275$ , and  $\lambda = \sin \theta_1 \cos \theta_3 = 0.2253$ . The determinant can be  $m_u m_c m_t$  with (31), (22), and (13) elements for the orders given above. So, we take (11), (12), and (21) elements with inequality signs.

Before presenting a numerical study, let us check that solutions suggested in Eqs. (28–36) are possible. From the (23) element, we restrict  $s'_2$  and  $s'_3$  at order  $\lambda^2$ ,

$$\begin{aligned} (23) : m_u c'_3 s'_1 s_1 s_3 + m_c [c'_1 c'_2 c'_3 + s'_2 s'_3 e^{i\delta'_{\text{CKM}}}] \cdot [c_1 c_2 s_3 - s_2 c_3 e^{-i\delta_{\text{CKM}}}] \\ + m_t [-c'_2 s'_3 + c'_1 s'_2 c'_3 e^{-i\delta'_{\text{CKM}}}] \cdot [c_2 c_3 + c_1 s_2 s_3 e^{i\delta_{\text{CKM}}}] \simeq 0 \\ \rightarrow s'_2 = O(\lambda^2), s'_3 = O(\lambda^2). \end{aligned} \quad (37)$$



Then, we satisfy (32) and (22) elements,

$$(32) : m_u s'_1 s'_3 s_1 c_3 + m_c [c'_1 c'_2 s'_3 - s'_2 c'_3 e^{i\delta'_{\text{CKM}}}] \cdot [c_1 c_2 c_3 + s_2 s_3 e^{-i\delta_{\text{CKM}}}] \\ + m_t [c'_2 c'_3 + c'_1 s'_2 s'_3 e^{-i\delta'_{\text{CKM}}}] \cdot [-c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\text{CKM}}}] = O(\lambda^2), \quad (38)$$

$$(22) : m_u c'_3 s'_1 s_1 c_3 + m_c [c'_1 c'_2 c'_3 + s'_2 s'_3 e^{i\delta'_{\text{CKM}}}] \cdot [c_1 c_2 c_3 + s_2 s_3 e^{-i\delta_{\text{CKM}}}] \\ + m_t [-c'_2 s'_3 + c'_1 s'_2 c'_3 e^{-i\delta'_{\text{CKM}}}] \cdot [-c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\text{CKM}}}] = O(\lambda^2). \quad (39)$$

Now, the (12) element restricts  $s'_1$  at order  $\lambda^2$ ,

$$(12) : m_u c'_1 s_1 c_3 - m_c c'_2 s'_1 [c_1 c_2 c_3 + s_2 s_3 e^{-i\delta_{\text{CKM}}}] - m_t s'_1 s'_2 e^{-i\delta'_{\text{CKM}}} \cdot [-c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\text{CKM}}}] = O(\lambda^4) \\ \rightarrow s'_1 = O(\lambda^2) \quad (40)$$

Then, the (11) element is very small,  $O(\lambda^5)$ . The remaining (21), (13), and (31) elements are

$$(21) : m_u c'_3 s'_1 c_1 - m_c [c'_1 c'_2 c'_3 + s'_2 s'_3 e^{i\delta'_{\text{CKM}}}] c_2 s_1 - m_t [-c'_2 s'_3 + c'_1 s'_2 c'_3 e^{-i\delta'_{\text{CKM}}}] \cdot s_1 s_2 e^{i\delta_{\text{CKM}}} = O(\lambda^3), \quad (41)$$

$$(13) : m_u c'_1 s_1 s_3 - m_c c'_2 s'_1 [c_1 c_2 s_3 - s_2 c_3 e^{-i\delta_{\text{CKM}}}] - m_t s'_1 s'_2 e^{-i\delta'_{\text{CKM}}} \cdot [c_2 c_3 + c_1 s_2 s_3 e^{i\delta_{\text{CKM}}}] = O(\lambda^4), \quad (42)$$

$$(31) : m_u s'_1 s'_3 c_1 - m_c [c'_1 c'_2 s'_3 - s'_2 c'_3 e^{i\delta'_{\text{CKM}}}] c_2 s_1 - m_t [c'_2 c'_3 + c'_1 s'_2 s'_3 e^{-i\delta'_{\text{CKM}}}] \cdot s_1 s_2 e^{i\delta_{\text{CKM}}} = O(\lambda^3) \quad (43)$$

where we considered  $m_c = O(\lambda^2)m_t$ . Here the rough bound of Eqs. (28–36) are satisfied except in Eq. (41). But,  $m_c$  is between  $O(\lambda^2)m_t$  and  $O(\lambda^3)m_t$  and Eqs. (41) is acceptable in our rough estimation. In our order of estimation,  $\delta'_{\text{CKM}}$  is not restricted.<sup>5</sup>

Therefore, the mass matrices Eqs. (17) and (18) obtained from anti-SU(7) UGUTF leads to a reasonable CKM matrix. Similarly, one can consider the lepton mixing angles which however need singlet contributions. Since there will appear additional parameters for the unknown heavy neutral lepton masses, there will be more freedom fitting for a reasonable PMNS matrix [37].

#### IV. BOUNDS ON THE PARAMETERS OF RIGHT-HANDED UNITARY MATRIX $V_R^{(u)}$

In Fig. 2, we present the allowed angles of  $V_R^{(u)}$ . The color code is: the projection on  $\theta'_2$  versus  $\theta'_1$  for all allowed  $\theta'_3$  and  $\delta'_{\text{CKM}}$  as blue, and  $\delta'_{\text{CKM}}$  versus  $\theta'_3$  for all allowed  $\theta'_1$  and  $\theta'_2$  as red. We allowed the  $1\sigma$  for  $\theta_1, \theta_2, \theta_3$  and  $\delta_{\text{CKM}}$  in  $V_L^{(u)}$ . We choose the  $V_L$  angles as  $\theta_1 = 13.025^\circ_{-0.038^\circ}^{+0.039^\circ}$ ,  $\theta_2 = 2.292^\circ_{-2.217^\circ}^{+2.625^\circ}$ ,<sup>6</sup>  $\theta_3 = 8.8923^\circ_{-0.0357^\circ}^{+0.0382^\circ}$ , and  $\delta_{\text{CKM}} = 85.4^\circ_{-3.8^\circ}^{+3.9^\circ}$  [2]. For the right-hand sides (of equality or inequalities) in Eqs. (28–28), the expansion parameter  $\lambda^n$  is varied in the region  $a^n \leq \lambda^n \leq b^n$ . In Fig. 2 (a), we choose  $a = \frac{2}{3} \sin \theta_C$  and  $b = \frac{3}{2} \sin \theta_C$ . In Fig. 2 (b), we choose  $a = \frac{1}{1.2} \sin \theta_C$  and  $b = 1.2 \sin \theta_C$ . From Fig. 2, we conclude that the mass matrices Eqs. (17) and (18), suggested from the UGUTF anti-SU(7), are phenomenologically allowed.

#### V. CONCLUSION

We presented bounds on the mixing angles of the right-handed currents, diagonalizing the quark mass matrices, suggested from a recently proposed families unified GUT model based on anti-SU(7) [1]. The investigation suggests that quark mass matrices suggested in [1] are phenomenologically allowable, and a numerical search is presented in figures on four mixing angles of  $V_R^{(u)}$  within the  $1\sigma$  bounds of the CKM parameters,  $\theta_1, \theta_2, \theta_3$ , and  $\delta_{\text{CKM}}$ . Along the way, we also commented on some aspects of the Jarlskog determinant. In particular, the currently allowed CKM parametrization falls into three classes for choosing  $\delta_{\text{CKM}} = \alpha, \beta$ , or  $\gamma$  of the PDG book. The Kobayashi-Maskawa and Kim-Seo parametrization choose  $\delta_{\text{CKM}} = \alpha$  and Chau-Keung-Maiani parametrization chooses  $\delta_{\text{CKM}} = \gamma$ . It suggests that with three real CKM angles fixed, the Jarlskog determinant is maximum with  $\alpha = \frac{\pi}{2}$ .

<sup>5</sup> In the numerical study below,  $\theta'_3$  is not bounded also.

<sup>6</sup> The lower limit is given from the measured value of  $J \simeq 10^{-5}$ .

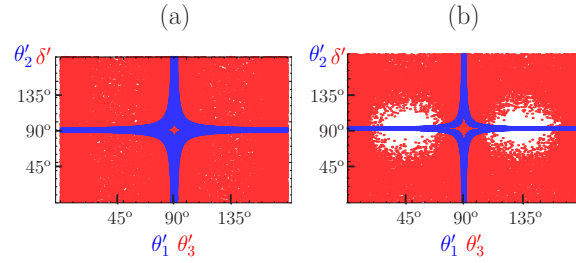


FIG. 2: The bounds on the angles of the right-handed unitary matrix  $V_R^{(u)}$  diagonalizing  $M^{(u)}$ , (a)  $a = \frac{1}{1.5} \sin \theta_c$ ,  $b = 1.5 \sin \theta_c$ , and (b)  $a = \frac{1}{1.2} \sin \theta_c$ ,  $b = 1.2 \sin \theta_c$ . The white regions are not allowed.

### Acknowledgments

We would like to thank Hyung Do Kim for useful communications. This work is supported in part by the National Research Foundation (NRF) grant funded by the Korean Government (MEST) (No. 2005-0093841) and by the IBS (IBS-R017-D1-2014-a00).

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- [1] J.E. Kim, *Towards unity of families: anti-SU(7) from  $\mathbf{Z}_{12-I}$  orbifold compactification*, JHEP **1506** (2015) 114 [arXiv:1503.03104 [hep-ph]].
  - [2] A. Ceccucci, Z. Ligeti, and Y. Sakai, *The CKM quark mixing matrix*, in K.A. Olive et al. (Particle Data Group), Chin.

Phys. C, 38, 090001 (2014).

- [3] N. Cabibbo, *Unitary symmetry and leptonic decays*, Phys. Rev. Lett. **10** (1963) 531.
- [4] M. Kobayashi and T. Maskawa, *CP violation in the renormalizable theory of weak interaction*, Prog. Theor. Phys. **B49**, 652 (1973).
- [5] L. Wolfenstein, *Parametrization of the Kobayashi-Maskawa matrix*, Phys. Rev. Lett. **51** (1983) 1945 [doi: 10.1103/PhysRevLett.51.1945].
- [6] L.-L. Chau and W.-Y. Keung, *Comments on the parametrization of the Kobayashi-Maskawa matrix*, Phys. Rev. Lett. **53** (1984) 1802 [doi: 10.1103/PhysRevLett.53.1802]. Multiplying an overall phase, see, L. Maiani, *CP violation in purely lefthanded weak interactions*, Phys. Lett. B **62** (1976) 183 [doi: 10.1016/0370-2693(76)90500-1] and *New currents*, Proc. 1977 Int. Symp. on Lepton and Photon Interactions at High Energies (DESY, Hamburg, 1977), p. 867.
- [7] J. E. Kim and M.-S. Seo, *Parametrization of the CKM matrix*, Phys. Rev. D **84** (2011) 037393 [arXiv:1105.3304 [hep-ph]].
- [8] C. Jarlskog, *Commutator of the quark mass matrices in the standard electroweak model and a measure of maximal CP violation*, Phys. Rev. Lett. **55** (1985) 1039 [doi: 10.1103/PhysRevLett.55.1039]; C. Jarlskog, *A basis independent formulation of the connection between quark mass matrices, CP violation and experiment*, Z. Phys. **C29** (1985) 491 [doi: 10.1007/BF01565198].
- [9] F. Wilczek and A. Zee, *Horizontal interaction and weak mixing angles*, Phys. Rev. Lett. **42** (1979) 421 [doi: 10.1103/PhysRevLett.42.421].
- [10] S. F. King and G. G. Ross, *Fermion masses and mixing angles from  $SU(3)$  family symmetry*, Phys. Lett. B **520** (2001) 243 [arXiv:hep-ph/0108112].
- [11] K. Choi, E. J. Chun, K. Hwang, and W. Y. Song, *Bimaximal neutrino mixing and small  $U_{e3}$  from Abelian flavor symmetry*, Phys. Rev. D **64** (2001) 113013 [arXiv: hep-ph/0107083].
- [12] S. Pakvasa and H. Sugawara, *Discrete symmetry and Cabibbo angle*, Phys. Lett. B **73** (1978) 61; G. Segré and H. A. Weldon, *Natural suppression of strong P and T violations and calculable mixing angles in  $SU(2) \times U(1)$* , Phys. Rev. Lett. **42** (1979) 1191; P. F. Harrison, D. H. Perkins, and W. G. Scott, *Threefold maximal lepton mixing and the solar and atmospheric neutrino deficits*, Phys. Lett. B **349** (1995) 137 [doi: 10.1016/0370-2693(95)00213-5].
- [13] E. Ma and G. Rajasekaran, *Softly broken  $A_4$  symmetry for nearly degenerate neutrino masses*, Phys. Rev. D **64** (2001) 113012 [arXiv: hep-ph/0106291]; G. Altarelli, F. Feruglio, and C. Hagedorn, *SUSY  $SU(5)$  grand unified model of tri-bimaximal mixing from  $A_4$* , JHEP **0803** (2008) 052 [arXiv:0802.0090 [hep-ph]].
- [14] H. Ishimori and S. F. King, *A model of quarks with  $\Delta(6N^2)$  family symmetry*, Phys. Lett. B **735** (2014) 33 [arXiv:1403.4395 [hep-ph]].
- [15] A. Blum, C. Hagedorn and M. Lindner, *Fermion masses and mixings from dihedral flavor symmetries with preserved subgroups*, Phys. Rev. D **77** (2008) 076004 [arXiv:0709.3450]; J. E. Kim and M.-S. Seo, *Quark and lepton mixing angles with a dodeca-symmetry*, JHEP **1102** (2011) 097 [arXiv:1005.4684 [hep-ph]] and *Parametrization of PMNS matrix based on dodeca-symmetry*, Int. J. Mod. Phys. A **27** (2012) 1250017 [arXiv:1106.6117 [hep-ph]].
- [16] H. Georgi, *Towards a grand unified theory of flavor*, Nucl. Phys. B **156** (1979) 126 [doi: 10.1016/0550-3213(79)90497-8].
- [17] J. C. Pati and Abdus Salam, *Unified lepton-hadron symmetry and a gauge theory of the basic interactions*, Phys. Rev. D **8** (1973) 1240 [doi: 10.1103/PhysRevD.8.1240].
- [18] H. M. Georgi and S. L. Glashow, *Unity of all elementary particle forces*, Phys. Rev. Lett. **32** (1974) 438 [doi: 10.1103/PhysRevLett.32.438].
- [19] S. M. Barr, *A new symmetry breaking pattern for  $SO(10)$  and proton decay*, Phys. Lett. B **112** (1982) 219 [doi: 10.1016/0370-2693(82)90966-2].
- [20] J. P. Derendinger, J. E. Kim, and D. V. Nanopoulos, *Anti- $SU(5)$* , Phys. Lett. B **139** (1984) 170 [doi:10.1016/0370-2693(84)91238-3].
- [21] I. Antoniadis, J. R. Ellis, J. S. Hagelin, and D. V. Nanopoulos, *The flipped  $SU(5) \times U(1)$  string model revamped*, Phys. Lett. B **231** (1989) 65 [doi: 10.1016/0370-2693(89)90115-9], and references cited therein.
- [22] J. E. Kim and B. Kyae, *Flipped  $SU(5)$  from  $Z_{12-I}$  orbifold with Wilson line*, Nucl. Phys. B **770** (2007) 47 [arXiv: hep-th/0608086].
- [23] J. E. Kim and H. P. Nilles, *The  $\mu$  problem and the strong CP problem*, Phys. Lett. B **138** (1984) 150 [doi: 10.1016/0370-2693(84)91890-2]; J. A. Casas and C. Muñoz, *Natural solution to the  $\mu$  problem*, Phys. Lett. B **306** (1993) 288 [arXiv: hep-ph/9302227].
- [24] H. Baer, K.-Y. Choi, J. E. Kim, and L. Roszkowski, *Dark matter production in the early Universe: beyond the thermal WIMP paradigm*, Phys. Rep. **555** (2015) 1 [arXiv:1407.0017 [hep-ph]].
- [25] S. B. Giddings and A. Strominger, *Axion induced topology change in quantum gravity and string theory*, Nucl. Phys. B **306** (1987) 890 [doi:10.1016/0550-3213(88)90446-4]; G. Gilbert, *Wormhole induced proton decay*, Nucl. Phys. B **328** (1989) 159 [doi: 10.1016/0550-3213(89)90097-7].
- [26] S. M. Barr and D. Seckel, *Planck scale corrections to axion models*, Phys. Rev. D **46** (1992) 539 [doi: 10.1103/PhysRevD.46.539]; M. Kamionkowski and J. March-Russell, *Planck scale physics and the Peccei-Quinn mechanism*, Phys. Lett. B **282** (1992) 137 [hep-th/9202003]; R. Holman, S. D. H. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins, and L. M. Widrow, *Solutions to the strong CP problem in a world with gravity*, Phys. Lett. B **282** (1992) 132 [hep-ph/9203206]; B. A. Dobrescu, *The Strong CP problem versus Planck scale physics*, Phys. Rev. D **55** (1997) 5826 [hep-ph/9609221].
- [27] L. M. Krauss and F. Wilczek, *Discrete gauge symmetry in continuum theories*, Phys. Rev. Lett. **62** (1989) 1221 [doi: 10.1103/PhysRevLett.62.1221].
- [28] L. E. Ibanez and G. G. Ross, *Discrete gauge symmetries and the origin of baryon and lepton number conservation in supersymmetric versions of the standard model*, Nucl. Phys. B **368** (1992) 3 [doi: 10.1016/0550-3213(92)90195-H].

- [29] J. E. Kim, *Natural Higgs-flavor-democracy solution of the  $\mu$  problem of supersymmetry and the QCD axion*, Phys. Rev. Lett. **111** (2013) 031801 [arXiv:1303.1822 [hep-ph]]; *Abelian discrete symmetries  $\mathbf{Z}_N$  and  $\mathbf{Z}_{nR}$  from string orbifolds*, Phys. Lett. B **726** (2013) 450 [arXiv:1308.0344 [hep-th]].
- [30] J. E. Kim and H. P. Nilles, *Dark energy from approximate  $U(1)_{de}$  symmetry*, Phys. Lett. B **730** (2014) 83 [arXiv:1311.0012 [hep-ph]]; J. E. Kim, *Modeling the small dark energy scale with a quintessential pseudoscalar boson*, J. Korean Phys. Soc. **64** (2014) 795 [arXiv:1311.4545 [hep-ph]].
- [31] S. Hamidi and C. Vafa, *Interactions on orbifolds*, Nucl. Phys. B **279** (1987) 465 [doi: 10.1016/0550-3213(87)90006-X]; L. J. Dixon, D. Friedan, E. J. Martinec, and S. H. Shenker, *The Conformal Field Theory of Orbifolds*, Nucl. Phys. B **282** (1987) 13 [doi: 10.1016/0550-3213(87)90676-6].
- [32] K.-S. Choi and T. Kobayashi, *Higher order couplings from heterotic orbifold theory*, Nucl. Phys. B **797** (2008) 295 [arXiv: 0711.4894 [hep-th]].
- [33] J. E. Kim, D. Y. Mo, and S. Nam, *Final state interaction phases using data from CP asymmetries*, J. Korean Phys. Soc. **66** (2015) 894 [arXiv: 1402.2978 [hep-ph]].
- [34] T. D. Lee, *A theory of spontaneous T violation*, Phys. Rev. D **8** (1973) 1226 [doi: 10.1103/PhysRevD.8.1226].
- [35] J. E. Kim, *The CKM matrix with maximal CP violation from  $\mathbf{Z}_{12}$  symmetry*, Phys. Lett. B **704** (2011) 360 [arXiv: 1109.0995 [hep-ph]]. See, also, H. Georgi, A. E. Nelson, and M. Shin, *CP violation and Fritzsch mass matrices*, Phys. Lett. B **150** (1985) 306 [doi: 10.1016/0370-2693(85)91016-0]; M. Shin, *A natural theory of the KM mixing angles, weak and strong CP*, Phys. Lett. B **160** (1985) 411 [doi: 10.1016/0370-2693(85)90011-5].
- [36] T. W. B. Kibble, *Topology of cosmic strings and domains*, J. Phys. A **9** (1976) 1387 [doi: 10.1088/0305-4470/9/8/029].
- [37] B. Pontecorvo, *Mesonium and anti-mesonium*, JETP **6** (1957) 429 [Zh. Eksp. Teor. Fiz. **33**, 549 (1957)]; Z. Maki, M. Nakagawa and S. Sakata, *Remarks on the unified model of elementary particles*, Prog. Theor. Phys. **B28**, 870 (1962).
- [38] J. E. Kim and S. Nam, *Is  $\delta_{CKM} \simeq \pm \delta_{PMNS}$ ?*, arXiv:1506.08491 [hep-ph].